Unit V: Introduction to Game Theory

Game theory is a distinct and interdisciplinary approach to the study of human behavior. The disciplines most involved in game theory are mathematics, economics and the other social and behavioral sciences. Game theory (like computational theory and so many other contributions) was founded by the great mathematician John von Neumann.

Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcome results. Game theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected. Thus it is a decision theory useful in competitive situations.

Game is defined as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss. The set of rules defines the **game**. Going through the set of rules once by the participants defines a **play**.

A Scientific Metaphor

Since the work of John von Neumann, "games" have been a scientific metaphor for a much wider range of human interactions in which the outcomes depend on the interactive strategies of two or more persons, who have opposed or at best mixed motives. Among the issues discussed in game theory are

1) What does it mean to choose strategies "rationally" when outcomes depend on the strategies chosen by others and when information is incomplete?

2) In "games" that allow mutual gain (or mutual loss) is it "rational" to cooperate to realize the mutual gain (or avoid the mutual loss) or is it "rational" to act aggressively in seeking individual gain regardless of mutual gain or loss?

3) If the answers to 2) are "sometimes," in what circumstances is aggression rational and in what circumstances is cooperation rational?

4) In particular, do ongoing relationships differ from one-off encounters in this connection?

5) Can moral rules of cooperation emerge spontaneously from the interactions of rational egoists?

6) How does real human behavior correspond to "rational" behavior in these cases?

7) If it differs, in what direction? Are people more cooperative than would be "rational?" More aggressive? Both?

Thus, among the "games" studied by game theory are

- Bankruptcy
- Barbarians at the Gate
- Battle of the Networks
- Caveat Emptor
- Conscription
- Coordination
- Escape and Evasion
- Frogs Call for Mates
- Hawk versus Dove
- Mutually Assured Destruction
- Majority Rule
- Market Niche
- Mutual Defense
- Prisoner's Dilemma
- Subsidized Small Business
- Tragedy of the Commons
- Ultimatum
- Video System Coordination

Why Do Economists Study Games?

- Games are a convenient way in which to model the strategic interactions among economic agents.
- Many economic issues involve strategic interaction.
  - Behavior in imperfectly competitive markets, e.g. Coca-Cola versus Pepsi.
  - Behavior in auctions, e.g. Investment banks bidding on U.S. Treasury bills.
  - Behavior in economic negotiations, e.g. trade.
- Game theory is not limited to Economics

Properties of a Game

1. There are finite numbers of competitors called 'players'
2. Each player has a finite number of possible courses of action called 'strategies'
3. All the strategies and their effects are known to the players but player does not know which strategy is to be chosen.
4. A game is played when each player chooses one of his strategies. The strategies are assumed to be made simultaneously with an outcome such that no player knows his opponents strategy until he decides his own strategy.
5. The game is a combination of the strategies and in certain units which determines the gain or loss.
6. The figures shown as the outcomes of strategies in a matrix form are called 'pay-off matrix'.
7. The player playing the game always tries to choose the best course of action which results in optimal pay off called 'optimal strategy'.
8. The expected pay off when all the players of the game follow their optimal strategies is known as 'value of the game'. The main objective of a problem of a game is to find the value of the game.
9. The game is said to be 'fair' game if the value of the game is zero otherwise it s known as 'unfair'.

Characteristics of Game Theory

1. Competitive game

A competitive situation is called a **competitive game** if it has the following four properties

1. There are finite number of competitors such that $n \geq 2$. In case $n = 2$, it is called a **two-person game** and in case $n > 2$, it is referred as **n-person game**.

2. Each player has a list of finite number of possible activities.
3. A play is said to occur when each player chooses one of his activities. The choices are assumed to be made simultaneously i.e. no player knows the choice of the other until he has decided on his own.
4. Every combination of activities determines an outcome which results in a gain of payments to each player, provided each player is playing uncompromisingly to get as much as possible. Negative gain implies the loss of same amount.

2. Strategy

The strategy of a player is the predetermined rule by which player decides his course of action from his own list during the game. The two types of strategy are

1. Pure strategy
2. Mixed strategy

Pure Strategy

If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

Mixed Strategy

If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus the mixed strategy is a selection among pure strategies with fixed probabilities.

Repeated Game Strategies

- In repeated games, the sequential nature of the relationship allows for the adoption of strategies that are contingent on the actions chosen in previous plays of the game.
- Most contingent strategies are of the type known as "trigger" strategies.
- Example trigger strategies
  - In prisoners' dilemma: Initially play doesn't confess. If your opponent plays Confess, then play Confess in the next round. If your opponent plays don't confess, then play doesn't confess in the next round. This is known as the "tit for tat" strategy.

– In the investment game, if you are the sender: Initially play Send. Play Send as long as the receiver plays Return. If the receiver plays keep, never play Send again. This is known as the "grim trigger" strategy.

3. Number of persons

A game is called ‘n’ person game if the number of persons playing is ‘n’. The person means an individual or a group aiming at a particular objective.

Two-person, zero-sum game

A game with only two players (player A and player B) is called a ‘two-person, zero-sum game’, if the losses of one player are equivalent to the gains of the other so that the sum of their net gains is zero.

Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in a rectangular form.

4. Number of activities

The activities may be finite or infinite.

5. Payoff

The quantitative measure of satisfaction a person gets at the end of each play is called a payoff

6. Payoff matrix

Suppose the player A has ‘m’ activities and the player B has ‘n’ activities. Then a payoff matrix can be formed by adopting the following rules

- Row designations for each matrix are the activities available to player A
- Column designations for each matrix are the activities available to player B
- Cell entry V_{ij} is the payment to player A in A’s payoff matrix when A chooses the activity i and B chooses the activity j.
- With a zero-sum, two-person game, the cell entry in the player B’s payoff matrix will be negative of the corresponding cell entry V_{ij} in the player A’s payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

7. Value of the game

Value of the game is the maximum guaranteed game to player A (maximizing player) if both the players uses their best strategies. It is generally denoted by ‘V’ and it is unique.

Classification of Games

Simultaneous v. Sequential Move Games

- Games where players choose actions simultaneously are simultaneous move games.
  - Examples: Prisoners' Dilemma, Sealed-Bid Auctions.
  - Must anticipate what your opponent will do right now, recognizing that your opponent is doing the same.
- Games where players choose actions in a particular sequence are sequential move games.
  - Examples: Chess, Bargaining/Negotiations.
  - Must look ahead in order to know what action to choose now.
  - Many sequential move games have deadlines/ time limits on moves.
- Many strategic situations involve both sequential and simultaneous moves.

One-Shot versus Repeated Games

- One-shot: play of the game occurs once.
  - Players likely to not know much about one another.
  - Example - tipping on your vacation
- Repeated: play of the game is repeated with the same players.
  - Indefinitely versus finitely repeated games
  - Reputational concerns matter; opportunities for cooperative behavior may arise.
- Advise: If you plan to pursue an *aggressive* strategy, ask yourself whether you are in a one-shot or in a repeated game. If a repeated game, *think again*.

Generally games are classified into

- Pure strategy games
- Mixed strategy games

The method for solving these two types varies. By solving a game, we need to find best strategies for both the players and also to find the value of the game.

Pure strategy games can be solved by **saddle point method.**

The different methods for solving a mixed strategy game are

- **Analytical method**
- **Graphical method**
- **Dominance rule**
- **Simplex method**

Limitations of game theory

The major limitations are

- The assumption that the players have the knowledge about their own payoffs and others is rather unrealistic.
- As the number of players increase in the game, the analysis of the gaming strategies become increasingly complex and difficult.
- The assumptions of maximin and minimax show that the players are risk-averse and have complete knowledge of the strategies. It doesn't seem practical.
- Rather than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out collusion. Then the mixed strategies are not very useful.